REVIEWS

THREE PROBLEMS IN THE THEORY OF HEAT TRANSFER AND PHYSICAL HYDROGAS-DYNAMICS

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Introduction

The modern theory of convective heat transfer was first developed in the first quarter of the present century. In [1-4], the principles for the broad practical use of the similarity method for the analysis and formulation of experiments on heat transfer in flows of homogeneous media were laid down. The investigations of Taylor, Prandtl, Karman, and Nikuradze provided the basis for the effective semiempirical description of the mean turbulent flow in the vicinity of solid bodies. The basis for the development of statistical semiempirical theories of turbulent transfer was laid down in [6,7], although these are less well known than the analysis of turbulence given later in [8].

The experimental investigation of heat transfer in boiling at the physical level began with [9,10], while Styrikovich and Peterson began systematic experiments on the hydrodynamics of vapor—liquid mixtures. In [11], a generalizing analysis was given of the material then existing on the acting pressure head and relative phase velocities in the flow of a gas—liquid flux in tubes. In the 1930s, the first systematic experience in constructing two-velocity models of two-phase flows was obtained; in this context, note should be made of [12]. In the same period, a general analysis of the similarity conditions of thermohydrodynamic processes in phase transitions of the first kind was given [13].

In [14-18], heat-transfer problems were formulated for near- and supersonic gas-flow velocities, the concept of stagnation temperature, the temperature recovery coefficient, and the generalized heat-transfer coefficient calculated from the difference in the actual wall temperature and the stagnation temperature corresponding to the flow conditions were introduced.

The overall state of heat-transfer theory at the beginning of the 1940s is more completely reflected in [19,20].

By this stage, the three fundamental problems of physical thermogasdynamics — turbulence, a multiplicity of phases, and the effect of physicochemical transitions — had not only been perfectly clearly formulated, but had also been shown to be unusually complex and difficult to describe.

On the whole, experimental methods and the accumulation of qualitative experimental data on the thermohydrodynamics of multiphase systems was in an embryonic state at this time, and proved unequal to the task of solving the vast complex of applied and theoretical problems posed in the 1940s and 1950s by the appearance of nuclear power, space technology, and the vigorous development of large-scale chemical engineering. In theoretical terms, what was clear was the fundamental complexity of describing these global problems of the macrophysics of moving media, the role of similarity analysis, and the formulation of cycles of experimental research of a fundamental kind. However, there was an evident gap between the volume and programs of such work and the flow of experiments devoted to the search for answers to individual, very particular physical or technological questions.

On the other hand, many general problems of the theory of heat transfer and physical hydrodynamics had already, to a definite extent, been formulated, and a number of canonical relations had been determined in some form, either theoretically or in the form of generalized empirical dependences. The majority of new problems arising as a result of the internal development of science and in response to the requirements of engineering practice are more local in character, but they are by no means simple either in regard to the formulation of experiments or with respect to their analysis and generalization.

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All this leads to a diversity of approaches and proposed partial solutions, and sometimes also to the pursuit of blind alleys.

Of course the opinions expressed below are also the result of the activity of a definite scientific school and cannot be completely free from certain partial or even preconceived viewpoints, for which we ask, in advance, the reader's understanding and forgiveness.

We would like to thank M. A. Gol'dshtik, B. P. Mironov, V. E. Nakoryakov, and E. M. Khabakhasheva for assistance and discussion.

Turbulence at a Wall

A sufficiently complete idea of the development and current state of the theory and experimental investigation of turbulence may be obtained from [21-33], taken together.

Without dwelling on the stability problem of a laminar boundary layer, it will simply be noted that this problem is of interest both in a purely physicomathematical context and for its possible implications for methods of controlling boundary layers. However, attention should be drawn to two important new results: The development of a theory of self-oscillation improving the agreement between calculated and experimental values of the critical Reynolds number [32], and the ideas associated with strange attractors [34,35].

In the development of a turbulent boundary layer at a wall, there is inhomogeneity of the flow structure and of the field of its averaged and actual parameters transverse to the boundary layer. There may appear here an external noncorrespondence between the parameters of the averaged flow and the actual turbulence characteristics. Thus, in a viscous sublayer of the averaged flow the molecular friction is almost completely determined. At the same time, the degree of turbulence in the viscous sublayer is very significant and practically independent of the Reynolds number of the flow. At the core of the boundary layer, the degree of turbulence is of the order of the frictional coefficient, i.e., decreases with increase in Reynolds number.

The following properties and laws of the turbulent boundary layer at the wall may be regarded as firmly established.

1. The structure is laminar; five layers are distinctly distinguishable:

a) a viscous sublayer ($0 < y < y_1$) with an absolute predominance of the effect of molecular friction on the averaged flow

$$\tau \approx \mu \frac{\partial \langle u \rangle}{\partial y}, \tag{1}$$

a high degree of turbulence [36]

$$\sqrt{\langle u'^2 \rangle} \approx \frac{1}{3} \langle u \rangle, \tag{2}$$

and intense damping of turbulent viscosity and heat conduction in the direction toward the wall

$$\frac{\mu_T}{\mu} \sim \eta^3, \tag{3}$$

$$(\mathrm{Pr} \to \infty)\ \frac{\lambda_T}{\lambda} \sim \eta^4;\ \mathrm{Pr}_T \sim \frac{1}{\eta}, \tag{4}$$

the thickness of this layer for an incompressible liquid being of the order of

$$\eta_1 \approx 5-6; \tag{5}$$

b) an intermediate layer in which the molecular and turbulent friction in the averaged flow are comparable

$$y \to \bar{y}_1,\ \mu \gg \mu_T;\ y \to y_2,\ \mu \ll \mu_T. \tag{6}$$

For an incompressible liquid the external boundary of this layer is of the order of

$$\eta_2 \approx 30-50. \tag{7}$$

In the first approximation, the damping laws in Eqs. (3) and (4) may be extrapolated to this layer, with the introduction of exponential attenuation [37];

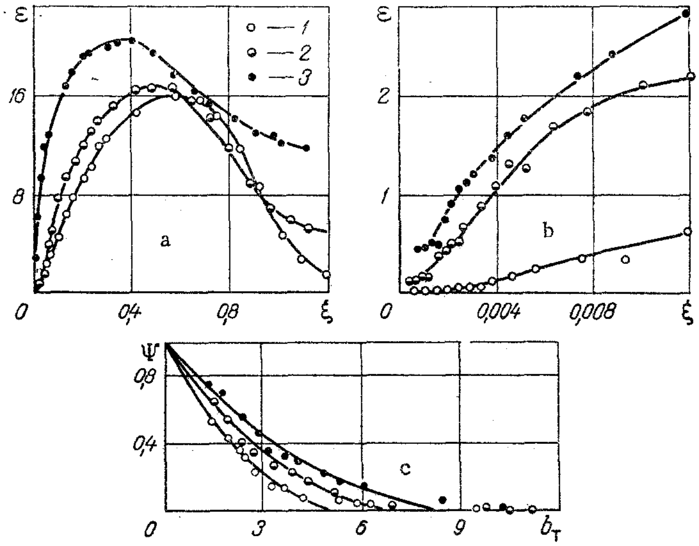

Fig. 1. Effect of external turbulence on the boundary-layer characteristics; a) intensity distribution of turbulent velocity-vector pulsations across boundary layer at a permeable surface; b ≈ 18; $\varepsilon_0 \sim 0.2$ (1), 5 (2), 11% (3); b) close to permeable surface; b = 18.2 (1), 19.4 (2), 16.3 (3); ε_0 = 0.1-0.3% (1), 3.6% (2), 11.4% (3); c) heat transfer at permeable surface; ε_0 = 0.2 (1), 5 (2), 15% (3).

c) the turbulent core, in which the main part of the dependence of the displacement path length on the transverse coordinate is linear (Prandtl law of the displacement path length)

$$y_2 < y < y_3;\ \mu_T \gg \mu;\ l \approx \varkappa y. \tag{8}$$

In the boundary layer of an incompressible liquid at a plate

$$\varkappa \approx 0.4;\ y_3 \approx 0.2\delta. \tag{9}$$

This value of $\varkappa$ is well-established experimentally, and is calculated theoretically in the quasi-laminar-stability model of turbulent flow in the immediate vicinity of a solid body [29,32];

d) the external turbulent region, in which the main part of the displacement path length of the averaged flow is proportional to the characteristic linear dimension of the boundary layer

$$y_3 < y < \delta;\ \mu_T \gg \mu;\ l \approx \varkappa_1 \delta. \tag{10}$$

The coefficients $\varkappa$ and $\varkappa_1$ may be related to one another by means of boundary conditions [29];

e) the mixing zone, leading to the region of unperturbed flow. The transfer laws in this zone have not yet been clearly investigated. They are significant for the diffuse exchange of impurities from the main unperturbed flow into the boundary layer but not greatly significant for the averaged flow at the wall. If the external flow has intrinsic turbulence, it exerts its influence on the flow at the wall through this layer (Fig. 1) [38,39].

2. Degeneracy effects of the averaged flow, the principal of which is expressed by the following relation [28,29]

$$\int_0^1 \left(\frac{\tilde{\rho}}{\Psi \tilde{\tau}_{\xi \ll 1}}\right)^{1/2} d\omega \xrightarrow[\mathrm{Re}\to\infty]{} 1. \tag{11}$$

3. Degeneracy effects of the thermal boundary layer flowing at an adiabatic surface [28,29]

$$\tilde{\delta}_T^{**} \xrightarrow[x_1 < x \to \infty]{} \int_0^1 \omega d\xi. \tag{12}$$

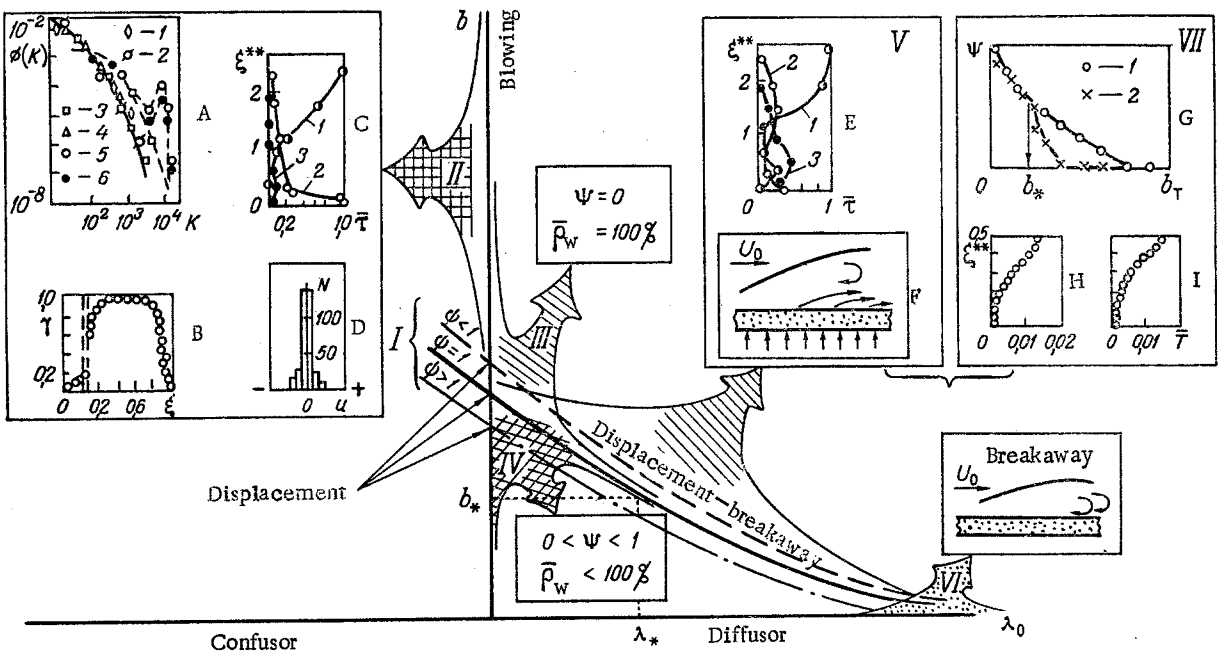

Fig. 2. Flow pattern of turbulent boundary layer at permeable surface: I) boundary at which critical blowing sets in, as a function of pressure gradient and nonisothermal conditions, $b_{cr} = f(\lambda_0, \psi)$; II) experimental characteristics of turbulent boundary layer with supercritical blowing, without pressure gradients; A, spectrum of turbulent velocity pulsations, b = 0–18, ξ = 0.01–0.72 (1–4), b → ∞, ξ = 0.0005 (5), b = 18.6, ξ = 0.0005 (6), K, m^{-1}; B, mixing-factor distribution over the boundary layer thickness; C, probability distribution of the flow direction over the boundary-layer thickness, for translational flow (1), upward flow (2), and backward flow (3); D, histogram of instantaneous values of the longitudinal velocity component in the immediate vicinity of the wall, b = 17.2, ξ = 0.003; III, IV) range of variation of Ψ and ρ_W for supercritical blowing, respectively; V) experimental characteristics of flow at the wall, supercritical blowing with a positive pressure gradient, displacement–breakaway; E, probability distribution of the flow directions over the boundary-layer thickness for translational (1), upward (2), and breakaway(3) flow; F, flow pattern for displacement–breakaway; VI) breakaway-flow region; VII) experimental characteristics of thermal boundary layer in the presence of blowing and positive pressure gradient; G, heat transfer for blowing and diffusor flow, with $\lambda_0 = 0$ (1) and $\lambda_0 = \lambda_*$ (2); H) temperature profile close to the wall in displacement–breakaway conditions, $\lambda_0 = \lambda_*$; I) temperature profile close to the wall in displacement conditions, $\lambda_0 = 0$.

In practice it may be assumed that the use of Eqs. (8), (11), and (12) within the framework of integral momentum and energy relations leads to a relatively simple and effective engineering method of calculating the turbulent heat and mass transfer and friction at surfaces without strong aerodynamic curvature in the case of unperturbed initial conditions.

The flow pattern of the two-dimensional boundary layer on the solid wall (Fig. 2) clearly demonstrates the basic aspects of the problem under consideration. Two will be discussed here: the breakaway and displacement of the boundary layer from the wall and its "memory." Both problems have been acknowledged for more than forty years but, strange as it seems, the stream of experimental results has not yet yielded sufficiently systematic and unambiguous results. At the same time, the problem of "memory" is important not only in the classical case of nonequilibrium boundary layers, but also in the effect of displacement of a boundary layer from a permeable surface. In this case, a turbulent boundary layer suspended on a "sharp jet" must "remember" its origin, associated with the friction at the solid wall [39]. As regards the breakaway zone, results of fundamental importance have recently been obtained: it has been shown that the Prandtl–Karman constant ϰ close to the breakway zone is significantly variable (Fig. 3). The breakaway parameters are stable only with respect to the velocity profile corresponding to the minimum critical value of the form parameter H ≈ 2. The real values of this parameter reaches three with the onset of developed breakaway, and the relation between H and the aerodynamic curvature f is found to be a function of the flow history.

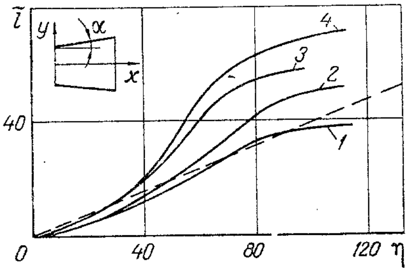

Fig. 3

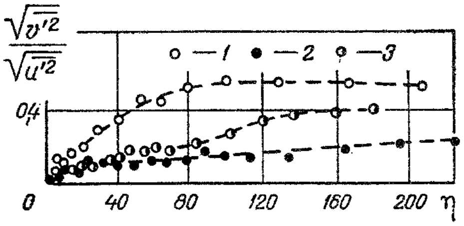

Fig. 4

Fig. 3. Dependence of Prandtl—Karman constants on aerodynamic curvature of surface. Flow of water in plane diffusor with angle of opening $\alpha = 4°$; form parameter H = 1.8 (1), 1.9 (2), 2.1 (3), 2.3 (4); dashed curve $\varkappa = 0.4$.

Fig. 4. Relation between different components of velocity pulsations in water and an aqueous solution of high-polymer: 1) water; 2) 0.07% polyethylene oxide solution; 3) 0.12% polyacrylamide solution.

It is expected that the extensive complex application of electrodiffusion tensometry, Doppler—Laser anemometry, and electronic stroboscopy will allow significant progress to be made in this research. Considerable possibilities are opened up by the large electrolytic hydrotube constructed at the Institute of Thermophysics.

Also of fundamental importance are the discovery of the asymmetric deformation of the pulsation-velocity components under the influence of high-polymer additions to a liquid medium flowing around a solid body (Fig. 4), the discovery of the influence of the thermophysical properties of the medium and the wall on the temperature pulsations and the turbulent Prandtl number in the immediate vicinity of a solid body [36,40-43]. The complexibility and present-day capabilities of statistical semiempirical theories of turbulence at a wall have been most completely outlined in [31]. Unfortunately, the experimental material on which such calculations are based is still limited. Therefore, study of the fine structure of the turbulent boundary layer in different situations is of particular importance, leading to an understanding of mechanisms affecting events over the whole thickness of the boundary layer but not emerging from an analysis of averaged characteristics. Such phenomena include the scatter of liquid nodes from the viscous sublayer in the external region of the boundary layer and the relation of these processes to turbulent-energy generation in the region at the wall.

It is found that some features of the averaged and pulsational flow may be qualitatively and, for some parameters, even quantitatively accounted for by considering a formally very simple monoharmonic model [32]. Thus, within the framework of this model, it is possible to construct the velocity field of a turbulent boundary layer of viscoelastic liquid that provides a good description of experiments with aqueous high-polymer solutions [44]. However, this approach is limited by the poorly predictable locality of the results, the complexity of the calculation, and the major uncertainty as to the existence of an efficient algorithms for successive approximation.

Turbulence at the wall in thermogravitational flows, the interaction of such flows with the induced motion of media, and the thermohydrodynamics of media at near-critical thermodynamic parameters relating to this circle of problems together form a singular and significant field within turbulence theory. Ultimately, the ideas and methods considered above retain their force in this field, in principle. However, there arise distinctive features that are often fundamental in character. Two examples will be given. In turbulent free thermal convection along a vertical plate, the flow divides significantly into an ordered viscous sublayer with an intrinsic Reynolds number which is a function of the physical Prandtl number of the medium, and an external flow analogous in form to a free submerged turbulent jet [45, 46].

With pronounced change in density (in the vicinity of the critical thermodynamic point), this effect begins to have an influence on the usually conservative turbulent characteristics, and must be taken into account. This subject has been considered in an extensive cycle

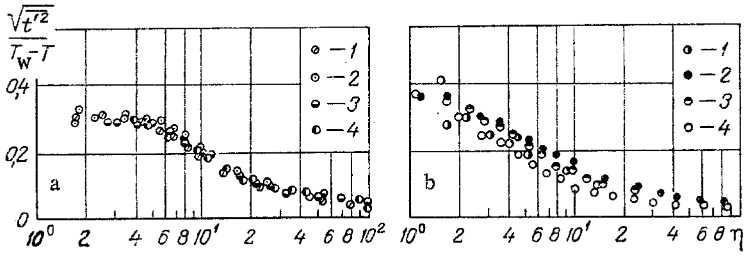

Fig. 5. Temperature pulsations in layer of incompressible liquid at wall: a) measurements at thick-walled copper surface, Re = 11500 (1), 21100 (2), 33000 (3), and 61000 (4); b) at a stainless-steel surface (thin tape), Re = 8200 (1), 20200 (2), 37000 (3), 53500 (4).

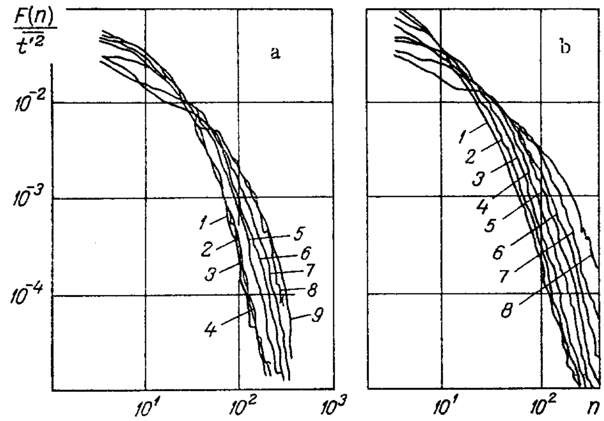

Fig. 6. Spectral density of temperature pulsations in flow, Re = $2 \cdot 10^4$: a) measurements at a copper plate, η = 1.6 (1), 2.7 (2), 3.5 (3), 4.8 (4), 7.4 (5), 11 (6), 16 (7), 44 (8), 47 (9); b) at a stainless-steel surface, η = 1.1 (1), 1.7 (2), 2.7 (3), 4.4 (4), 6.3 (5), 9.6 (6), 15 (7), 59 (8).

of experimental and theoretical work under the guidance of Petukhov [47,48].

The theoretical analysis of the formation of a turbulent boundary layer and breakaway effects in the flow in the vicinity of bodies in a weak flow (the canonical forms are a sphere and a cylinder transverse to the flow) has been relatively weakly developed. At the same time, the practical importance of such flows is extremely large, and some of their important properties have only been discovered comparatively recently [49,50].

Thermodynamics of Gas–Liquid Systems

The overall development of this problem over the last 30 years may be traced in [51-68]. The bibliographic guides [69,70] provide good information on recent publications. The technical applications of this field include energetics, chemical and petrochemical engineering, techniques of moderate and deep refrigeration, methods of cooling high-voltage radioelectronic systems and many others.

The most important aspects are: the multiplicity of flow structures and conditions; the existence of interactions at phase boundaries; phase transitions and metastable states; compressibility effects in systems of two incompressible components; wave effects and the influence of real phase compressibility even at small flow velocities; the multiplicity of acoustic characteristics; the effect of the geometry and physical properties of the centers of condensation of the emerging phase; the physicochemical properties of the contact surfaces (e.g., their wetting by liquid phase); the absence in many cases of an influence of primary events of new-phase generation, but the strong influence of local independence in the averaged flow.

In scientific terms, the main difficulties are associated with the absence of a single physicomathematical description, in particular, sufficiently inclusive methods of averaging the parameters of the gas—liquid flow, and with the absence, until recently, of effective methods of measuring the local actual and averaged values of the phase velocities, their enthalpies, tangential stresses, and concentrations. This problem, which is fundamental for the development of experimental investigations, has now been removed by the introduction of electrochemical and laser anemometry and tensometry. Modern computational methods based on multivelocity (usually two or three) model equations require clear experimental data on the intensity of momentum transfer, masses, and energies inside single-phase regions and at phase boundaries and also on the dynamic phase distribution function. It is impossible to find a universal dependence by this means. The strong and weak points of these generally useful and essential methods are evident, e.g., in [71,72]. The limited number of engineering methods of calculating specific objects differ significantly from country to country (for example, methods of calculating in-boiler hydrodynamics in the USSR and the USA). Therefore, methods of similarity analysis and comparison for the sufficiently general and effective physicomathematical models of the most important phenomena in the thermodynamics of gas—liquid systems are of exceptional importance both in theoretical analyses and in formulating and generalizing fundamental experimental research.

In general, the problem of analyzing similarity conditions in gas—liquid systems may be regarded as sufficiently clear. The main specific similarity criteria here are as follows: the structural-stability criterion

$$U''_{cr}\rho''^{1/2}/[g\sigma(\rho'-\rho'')]^{1/4};$$

the boundary viscous-stability criterion

$$U''/\sigma\mu';$$

the interaction criterion for viscous, surface and gravitational forces

$$\sigma^{3/2}\rho'^{1/2}/\mu'^2\,[g(1-\tilde{\rho}'')]^{1/2};$$

the phase-transition thermal criterion

$$r/\Delta i;$$

the wave-interaction criterion

$$[g\sigma/(\rho'-\rho'')]^{1/4}(\rho''/p)^{1/2}.$$

The characteristic internal linear scales are the Laplace constant $[\sigma/g(\rho'-\rho'')]^{1/2}$ and the order of the critical nucleation radius $\sigma T''/r\rho''\Delta T$.

The problem of wave-perturbation propagation in gas—liquid processes is central in most problems of the stability and intensity of heat and mass transfer. Thus, the effect of displacement of liquid from a microporous surface and its thermal analog — the first boiling crisis in nonviscous liquids — is determined by the dependence of the stability criterion k on the wave-interaction criterion M_*, while the heat-transfer intensity in bubble boiling is determined by the dependence

$$\mathrm{Nu}_* \sim (\mathrm{Pe}_*/M_*^2)^{2/3}. \tag{13}$$

Two problems will be considered as examples of more detailed analysis of wave effects in gas—liquid systems: wave propagation in a liquid with gas bubbles and wave formation at the surface of thin layers of viscous liquid. Both problems have fairly detailed histories, and acceptable solutions have only recently been projected. The study of wave processes in gas—liquid media was clearly proposed in [73], turning attention to the circumstance that sound is intensely damped in a bubble mixture and the perturbation propagation rate is considerably less than the velocity of sound even in a gas. In fact, introducing the mixture density

$$\rho_{mi}=\rho''\beta+\rho'(1-\beta)=\rho''[x^*+\tilde{\rho}''(1-x^*)]^{-1} \tag{14}$$

and differentiating with respect to p, the quantity

$$a=\left(\frac{dp}{d\rho_{mi}}\right)^{1/2}=\frac{x^*+(1-x^*)\tilde{\rho}''}{\sqrt{x^*\partial\rho''/\partial p+(1-x^*)\tilde{\rho}''^2\partial\rho'/\partial p}}, \tag{15}$$

which may be regarded as a measure of the sound velocity in the mixture, may be obtained. Passing to volume concentrations gives

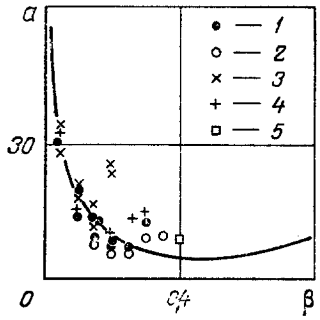

Fig. 7.

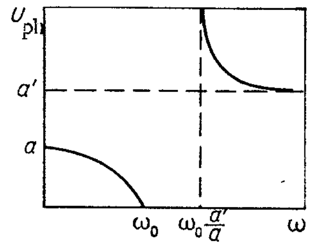

Fig. 8.

Fig. 7. Dependence of sound velocity on volume gas content in a quasi-homogeneous mixture (data of Campbell and Pitcher) at a flow velocity of 4.5 (1), 6 (2), 7.5 (3), 9 (4), and 12 (5) m/sec; a, m/sec.

Fig. 8. Dependence of phase velocity U_{ph} on frequency ω in liquid with uniformly distributed gas bubbles: a, low-frequency sound velocity; a', sound velocity in liquid.

$$a^2 = \tilde{\rho}''\left\{[1-\beta(1-\tilde{\rho}'')]\left[\beta\frac{\partial\rho''}{\partial p} + (1-\beta)\,\tilde{\rho}''\frac{\partial\rho'}{\partial p}\right]\right\}^{-1}. \tag{16}$$

It is not difficult to see that when $\tilde{\rho}''(a''/a')^2 \ll 1$ the minimum sound velocity is reached for $\beta = 1/2$. A curve of the sound velocity according to Eq. (16) is shown in Fig. 7.

However, the generalization of traditional mechanics to the case of the motion of liquid with gas bubbles runs into difficulty. Considering the linear acoustics of such a bubble mixture leads to the appearance of acoustic-wave dispersion, i.e. the sound velocity of the mixture at fixed gas content by weight begins to depend on the frequency (Fig. 8), which is associated with the inertial properties of the added mass of gas inclusions.

Important results on the propagation of perturbations in a liquid with gas bubbles are obtained on the basis of the Burger–Cortevega–de Fries equation

$$\frac{\partial\tilde{p}}{\partial H_0} + \tilde{p}\frac{\partial\tilde{p}}{\partial\xi_*} - \mathrm{Re}_*^{-1}\frac{\partial^2\tilde{p}}{\partial\xi_*^2} + \sigma_*^{-2}\frac{\partial^3\tilde{p}}{\partial\xi_*^3} = 0. \tag{17}$$

Thus, this equation takes into account the acoustic-wave dispersion (the initial portion on the left-hand branch of the dispersion curve), nonlinearity, and dispersion, due basically to three mechanisms: liquid viscosity, acoustic radiation of the gas bubbles, and heat losses taken into account through the effective viscosity $\eta*$ [73].

In particular, it is possible to establish that in a gas–liquid medium shock waves are monotonic and oscillating in structure, while the form of propagation of the perturbations is determined not only by the signal intensity but also by the initial width of the perturbation. In this system, there may exist individual waves (solitons), wave packets (wave trains made up of waves of varying sign), and shock waves with a monotonic or oscillating structure. The characteristic wave structures are shown in Fig. 9 in the coordinates $\sigma*$, $Re*$.

Further, the nontrivial role of bubble–liquid heat transfer and the appearance of thermal relaxation in shock waves has been established [74,75]. Thus, waves in a liquid with CO_2, N_2, and He bubbles have different structures at the same distance, and the oscillation depth at the shock wave front is considerably higher for CO_2 than in a liquid with N_2 and He bubbles. This corresponds to the abovementioned feature of the variation of the heat-transfer coefficient from a microporous surface to a liquid through which gas is bubbled.

A second problem is often associated with the name of P. L. Kapitsa, who in 1946 proposed the first model of wave formation at the surface of thin viscous-liquid layers [76]. To date, more than 100 papers of varying scientific level have appeared on this subject.

The problem of wave formation at a film is considered on the basis of the complete Navier–Stokes equations like the Orr–Sommerfeld stability problem for the case of a viscous liquid with an open upper boundary and in the boundary-layer approximation for a viscous

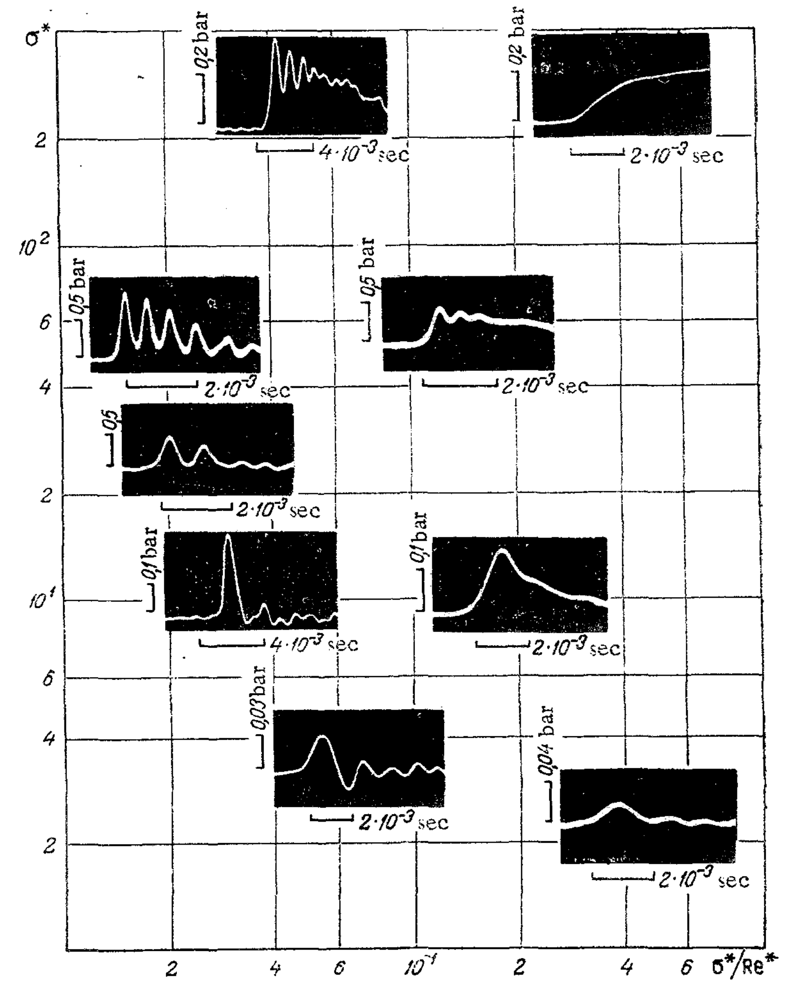

Fig. 9. Perturbation profiles in liquid with gas bubbles.

liquid [77-86]. Despite the well-known limitations of this approach, the results obtained on this basis overlap the results of Lin, Gjevik, and Maurin obtained from the complete Navier–Stokes equations. At the same time, the boundary-layer approximation offers a clear and lucid physical interpretation of the investigated waves. In particular, it is distinctly observed that, at the surface of a thin viscous-liquid layer, waves of two types exist: kinematic traveling over the surface of the film at a low-frequency velocity equal to $3U_0$ and described by the equation

$$\frac{\partial \varphi}{\partial H_0} + 3\frac{\partial \varphi}{\partial \tilde{x}} + \mathrm{Re}\frac{\partial^2 \varphi}{\partial \tilde{x}^2} + \mathrm{We}\frac{\partial^4 \varphi}{\partial \tilde{x}^4} = 0, \tag{18}$$

and also simple waves with a velocity of $1.7U_0$, described by the equation

$$\frac{\partial \varphi}{\partial H_0} + 1.7\frac{\partial \varphi}{\partial \tilde{x}} + 2.3\varphi\frac{\partial \varphi}{\partial \tilde{x}} - \mathrm{We}\frac{\partial^3 \varphi}{\partial \tilde{x}^3} - 3.9\frac{\varphi}{\mathrm{Re}} = 0. \tag{19}$$

The nonlinear two-wave equation containing both the kinematic and simple waves takes the form

$$\left(\frac{\partial}{\partial H_0} + 3\frac{\partial}{\partial \tilde{x}}\right)\varphi + \frac{\mathrm{Re}}{3}\left(\frac{\partial}{\partial H_0} + 1.69\frac{\partial}{\partial \tilde{x}}\right)\left(\frac{\partial}{H_0} + 0.71\frac{\partial}{\partial \tilde{x}}\right)\varphi + \mathrm{We}\frac{\partial^4 \varphi}{\partial \tilde{x}^4} + 6\varphi\frac{\partial \varphi}{\partial \tilde{x}} - \frac{2}{15}\mathrm{Re}\frac{\partial}{\partial H_0}\left(\varphi\frac{\partial \varphi}{\partial H_0}\right) = \tag{20}$$

In these equations Re is the ordinary Reynolds number of the film.

From the last equation, an individual-wave (soliton) solution may be obtained (Fig. 10), in which the amplitude increases with increase in liquid flow rate, but the thickness of the thin-liquid-layer supports under and between the waves remains practically the same. These thin viscous-liquid layers (and not the steep waves) are what determine the thermal resistance

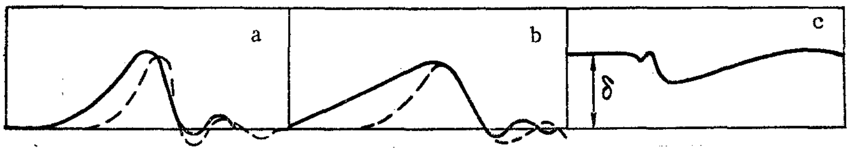

Fig. 10. Profiles of individual waves at the surface of thin liquid films: the dashed curves correspond to individual-wave theory and the continuous curves to experiment; the wave velocity is $4.4U_0$ (a), $5.5U_0$ (b), and $2.1U_0$ (c).

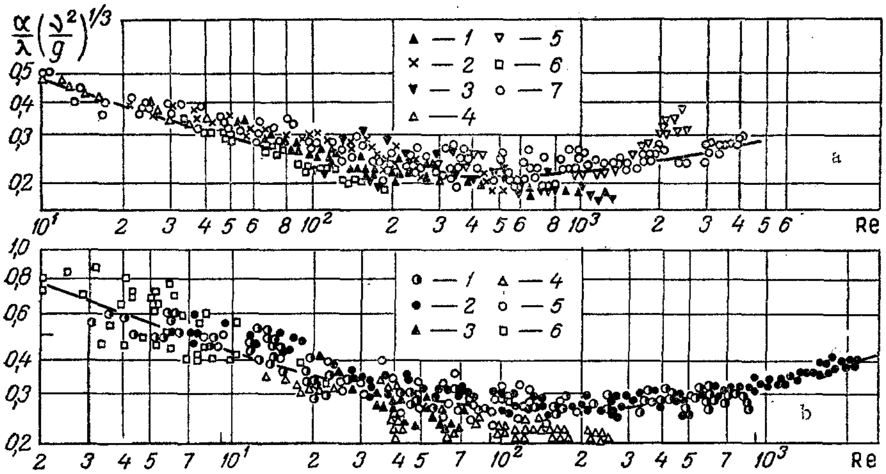

Fig. 11. Heat-transfer law for film condensation of pure saturated vapor: a) vertical surface, experimental results of Kutateladze and Shrentsel (1), Zozul (2), Meisenburg, Boart, and Bedzher (3), Ratiani and Shekriladze (4), Gudemchuk (5), Burov (6), and Gogonin, Dorokhov, and Sosunov (7); b) bundles of horizontal tubes, data of Gogonin and Dorokhov (1), Kutateledze, Gogonin, and Sosunov (2), Kutateledze (3), Isachenko and Glushkov (4), Yang and Volenburg (5), and Short and Braun (6). $Re = G/\nu$.

in film condensation over a broad region of the quasi-self-similar heat-transfer law, which is clearly observed both in condensation at vertical surfaces and in condensation at bundles of horizontal tubes (Fig. 11).

The problem of homogeneous condensation, especially in strongly nonequilibrium conditions, is of great interest in scientific terms and has ever expanding technological promise (e.g., in powder metallurgy). Intense gas expansion may occur with enormous supersaturation and accompanying spontaneous condensation, when the time to establish the size distribution of subcritical-scatter nuclei is less than the characteristic gasdynamic time. In this case, nuclei of critical size may have structural and thermophysical properties which differ from the large-volume condensed phase. Nevertheless, recent experiments show that the classical theory of homogeneous condensation works satisfactorily if the empirical constants of the theory (e.g., the surface tension of small nuclei and clusters) are consistent with experimental data.

Investigations on molecular-beam equipment, for example, on the large molecular-beam generator of the Institute of Thermophysics, often serve as such reference experiments. Characteristic results of molecular-beam measurements — beam intensity, monomer, dimer, trimer, and tetramer density — as a function of the pressure in the gas source at given stagnation temperature are shown in Fig. 12. The characteristic points on the curves correlate in some way with the form of gas condensation and are used to match classical theory and experiment, and also to establish empirical dependences for the calculation of condensation. The small-cluster intensity distribution on the section before the maximum may be used to determine the formation rate constants [87]. This problem has already been successfully solved for dimers. The control of cluster systems and the use of their interactions with the gas flow and with radiation represent very promising problems.

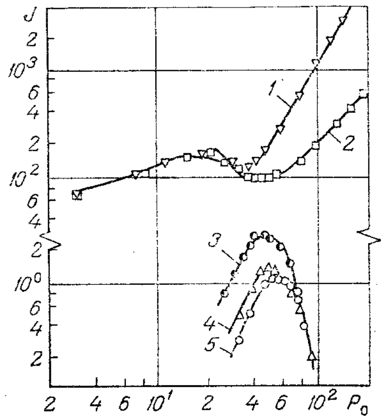

Fig. 12. Curves of CO_2 condensation in a jet: 1) CO_2-beam intensity; 2) monomer density; 3) dimer density; 4) trimer density; 5) tetramer density; J, arb. units; P_0, mm Hg.

The initial and established boiling of liquid in the wall layer of a flow with a core temperature less than the saturation temperature at the given pressure are also significantly nonequilibrium processes. There arise intense cavitation effects, which may affect, in particular, the state of surface heating. The picture regarding the effect of underheating of the liquid-flow core on the first boiling crisis is clear. However, the material available on bubble boiling and nonequilibrium effects for flow in channels is still very limited.

Crystallization processes are among the most extensive and important regions of nonequilibrium thermodynamics. Intense investigation of these processes began in the 1950s [88,89]. The essential point distinguishing the physical reality from the classical Stefan problem is the appearance of instability of the liquid—solid boundary. The appearance of deformation leads to the formation of a boundary with a cellular structure, at which inhomogeneous impurity (for example, doping additives) segregation occurs. This, in turn, leads to inhomogeneity of the material's physicochemical properties. However, segregation may be used, in principle, to obtain cast composite materials obtained directly in the crystallization process. The existing physicomathematical models proposing the presence of thermodynamic equilibrium of the phase boundary [90] are clearly unsatisfactory when it is impossible to neglect the formative influence of the concentration supercooled melt. One of the possible fruitful directions is the formulation of the problem of finding the form of the phase boundary, which within the framework of the nonequilibrium kinetic law adopted would be consistent with the intrinsic supercooling field. This approach, developed in the Institute of Thermophysics, has already allowed the problem of cellular-structure formation of crystalline materials to be solved.

Thermodynamics of Disperse Systems

The theoretical and applied importance of disperse systems of the type of a liquid or gas and solid particles has increased steadily in recent years, in connection with the requirements of nuclear and chemical engineering. Apparatus with granular motionless or fluidized layers has a very developed exchange surface and high values of the heat and mass transfer coefficients both from a particle to the flow and from the layer as a whole to the wall. This allows highly boosted equipment for very different purposes to be designed. There have been major successes in investigating granular systems. Theories of linear and nonlinear filtration have been developed, forming a large independent branch of liquid and gas mechanics. An enormous number of works have described the theoretical and experimental investigation of fluidized and vibrofluidized beds. Phenomenological and kinetic approaches involving statistical methods have been developed. The state of the engineering and physical investigations in this region may be judged from [91-96]. At the same time, adequate physical models capable of guiding the direction of experimental research have only recently appeared.

Some of the problems considered at the Institute of Thermophysics will now be touched upon. Classical filtration theory does not work in considering the problem of liquid inflow and outflow at the boundary of a porous medium. For example, the problem of liquid flow past

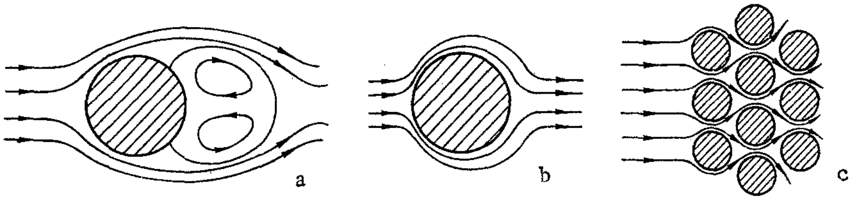

Fig. 13. Flow diagrams in the vicinity of a sphere: a) flow around a single sphere in an infinite space at sufficiently large Reynolds number; b) flow around a single sphere by a small-diameter jet; c) effective averaged flow in an array of spheres.

a porous insert in a channel has a multiplicity of solutions in the classical formulation. The quasi-ideal-liquid model, taking the action of inertial forces into account, allows reasonable boundary conditions to be formulated at the boundary of the porous medium, ensuring that the flow problem is uniquely solvable. The corresponding analysis reveals the formation of eddies at the boundary and at points of inhomogeneity of the medium, which cannot be described within the framework of Darcy's law.

The quasi-ideal-liquid model also allows the problem of the thermal stability of a heat-liberating granular bed cooled by a penetrating gas flow to be solved. It is found that, for sufficiently high levels of heat liberation, this system loses stability, and uncontrolled heating becomes possible. However, this model is very limited, and does not take into account, in particular, the deformation of the medium under the action of a sufficiently strong filtrational flux and the inverse effect of these deformations on the flow. This influence, as observations show, may be sufficiently significant, leading, for example, to the formation of sharp velocity-field inhomogeneities behind the bed. In general, the problem of constructing a random installation even of identical spheres has so far been lacking in theoretical analysis. Thus, the well-known experimental value of the volume concentration of dense packing, ≈ 0.6, is not confirmed by theory. It is usually assumed that the hydraulic drag of a bed of spheres depends only on their volume concentration. However, a number of data point to the influence of other factors associated with the order in the random packing, for which no adequate characteristics of theoretical description have been developed as yet. The problem of determining the deformed state of the bed under the action of applied forces has only very recently undergone development [97]. Until then, only a theory of the limiting equilibrium of a free-flowing medium, with very narrow limits of application, had been developed. It could only predict the slip surface in equilibrium stability loss, without saying anything about the prelimiting states.

The question of flux energy-transfer mechanisms by the randomly moving particles is central to the problem of a boiling or fluidized bed. The consideration of an elementary model of a boiling bed as a pseudogas of particles obtaining energy as a result of Magnus forces associated with particle rotation in the course of their flow is found to be effective [98,99]. For sufficiently concentrated systems, the rotation is generated as a result of particle impact on other particles and on the wall. These impacts are also the main mechanism of energy dissipation. Energy balance allows the chaotic particle velocities and their kinetic pressure to be calculated. The kinetic pressure is found to be a nonmonotonic function of the volume concentration of particles, with a maximum at a concentration of ≈ 0.35. This value determines the lower limit of stability of the concentrated suspended bed. Overall, the concentration dependence of the pressure resembles the saturation curve for actual liquids. This theory of a fluidized bed contains the description of two phases: the liquid and vapor phases, which are in equilibrium. The concentration 0.35 is critical. In ordinary fluidized beds, a high concentration exceeding 0.35 corresponds to only small initial fluidization number. As a rule, in practice, with the aim of boosting the process, the flow velocity is increased, and more attenuated beds are used, displaying instability in the form of bed pulsations, the appearance of gas breakthrough, bubbles, and particle fragments. Moreover, there is the possibility of retaining bed stability and homogeneity with arbitrarily high boosting if the gravitational fluidized bed is replaced by one that is centrifugal. As experiments have shown, the hydraulic drag of this bed is lower, and the heat transfer with the flow higher, than in an ordinary boiling bed. In addition, there is no sign of the stagnant no-flow regions characteristic of motionless beds. In such a bed, the particles do not

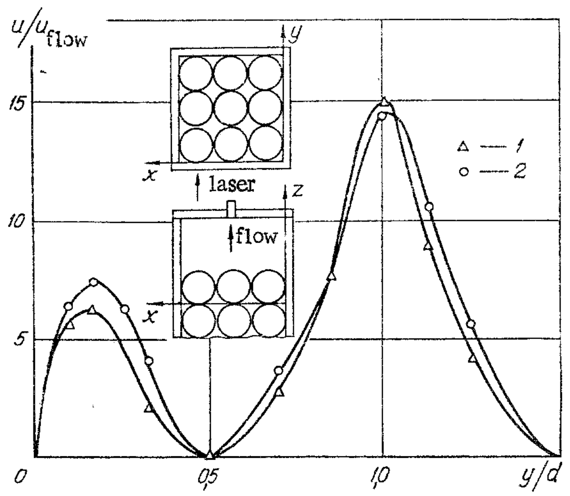

Fig. 14. Longitudinal-velocity distribution in narrow cross section of a regular cubic array of 9 spheres of diameter 18 mm over the channel cross section and 6 spheres over the channel height for Reynolds numbers constructed from the flow velocity in the free cross section and the sphere diameter as follows: Re = 25 (1), 70 (2); $z/d = -0.5$; $x/d = 1$.

touch one another and flow in some sense independently. This property is due to the random character of the particle flow in a high-concentration bed, when the equivalent diameter of the gap between particles becomes less than their size. As is known from experiment, spheres around which flows a jet of diameter less than the sphere diameter have two properties: first, the flow around them will be without breakaway regions; and second, the flow will be free and stably maintained about the jet axis. The first property ensures low hydraulic drag of the sphere with increased immersion of its surface; the second creates the preconditions for self-stabilization of the whole system by the formation of a quasicrystalline bed structure. The cases of a sphere in an infinite flow, in a jet flow, and in a centrifugal bed are shown in Fig. 13. In the latter two cases, in contrast to the first, reduction of hydraulic drag is observed, which is an additional valuable attribute of the quasicrystalline structure. Of course, the model of a pseudogas of particles is unsuitable for the description of such a structure, but it gives the bed stability limits correctly. It may also be used to estimate the stability of an attenuated bed in a swirling flow, for example, of eddy sink type. As analysis shows, an individual particle in such a flow may rotate stably about an equilibrium orbit. With rise in concentration, however, the field of azimuthal velocities is distorted, approaching the solid-rotation law, and in this case, as may readily be demonstrated, the equilibrium orbit is no longer stable. Hence, there is an upper limit on the concentration, below which an attenuated bed is stable. Thus, a centrifugal bed may exist in two stable states: attenuated and concentrated. They are divided by an instability barrier.

The models of an immobile array and a pseudogas of particles are only extreme cases of the possible states of a disperse medium, between which there is a whole spectrum of other states which have not yet been investigated. Nonsteady processes of a different kind have also been poorly investigated in disperse systems: wave propagation and the generation of self-oscillation and turbulence, where this phenomenon occurs in concentrated disperse systems. An important role is played by questions of the flow around bodies placed in moving fluidized beds and also questions of heat transfer. The development of experimental methods for the internal diagnostics of disperse systems if fundamental here. At present, the use of Doppler–laser anemometry and electrodiffusion tensoanemometry for the study of averaged and actual liquid motion inside a disperse bed and frictional distribution over the surface of solid particles is particularly promising.

As an illustration, the results of measuring the liquid velocity profile in a sphere-array cell are shown in Fig. 14. As is evident, the real flow is by no means adequately represented by an elementary interpretation of the results of integral measurements.

Some Engineering and Physics Problems

The abovementioned physical problems have numerous fields of application, some of them already noted, which involve long-term and multipurpose programs (e.g., methods of reducing the hydrodynamic drag, the internal thermodynamics of vapor generators, heat-transfer intensification in the condensation of refrigerants, aerodynamic protection of surfaces from the aggressive effects of the main flow of a working body, tempering metallic parts, etc.). The aim of this section is to direct attention to some specific problems requiring profound engineering and physical investigation, not only seeking constructional solutions but also finding regions of more effective technological applications:

1) the solution of thermophysical problems determining the economic efficiency of the use of superconductivity in developing electrical machines for various purposes and control equipment for high-power electrical systems;

2) the determination of regions of application and specific constructional solutions for eddy chambers with controlled thermohydrodynamics taking into account continuous flow conditions, eliminating wear or operating in wear conditions where this is necessary, separation, and division;

3) the creation of powerful flow-through cryovacuum systems for gas separation, condensation to the liquid and solid microdisperse state, the control of the external thermodynamics when radiation interacts with the condensed material;

4) the development of principles of aerodynamic control of production lines in promising methods of preelevator separation and treatment of granular piles;

5) the creation of operating unipolar electrical machines with liquid-metal high-power current pickups for electrometallurgy, shipbuilding, physical-research complexes, and other purposes.

NOTATION

Similarity parameters: $Ar_* = \sigma^{3/2}\rho'^{1/2}/\mu'^2[g(1 - \tilde{\rho}'')]^{1/2}$; $H_0 = tu_0/l_0$; $K = r/\Delta i$; $k = U''_{cr}\rho''^{1/2}/[g\sigma(\rho' - \rho'')]^{1/4}$; $M_* = (\rho''/\rho)^{1/2}[g\sigma/(\rho' - \rho'')]^{1/4}$; $Nu_* = \alpha\sigma^{1/2}/\lambda'[g(\rho' - \rho'')]^{1/2}$; $Pe_* = qc'\sigma^{1/2}/r\lambda'[g(\rho' - \rho'')]^{1/2}$; $Pr = \mu'c'/\lambda'$; $\tilde{p} = p'/p'_0$; $Re = -UL/\nu'$; $Re_* = u_0 l_0/\eta_*$; $We = \sigma/\rho V^2\delta_0$; $\eta = v_* y/\nu'$; $\xi_* = (x - c_0 t)/l_0$; $\sigma_* = u_0 l_0/\alpha_*$; a, sound velocity; c', liquid-phase specific heat; g, acceleration; δ_0, mean film thickness; $\delta_1 = \delta - \delta_0$, surface perturbation; δ, boundary-layer thickness, film thickness; L, characteristic linear dimension; l, displacement path length; $\tilde{l} = lv^*/y$; l_0, width of initial perturbation; p, pressure; p', pressure perturbation; p'_0, initial pressure perturbation; q, heat-flux density; r, latent heat of vaporization; U, characteristic flow velocity; U'', gas-phase (vapor-phase) velocity; U''_{cr}, critical gas-phase velocity; u, local longitudinal flow-velocity component; u', longitudinal pulsation-velocity component; v', transverse pulsation-velocity component; u_0, velocity perturbation; x, longitudinal coordinate; $\tilde{x} = x/\delta_0$; x_1, coordinate of adiabatic-section origin; x^*, gas content by weight; y, transverse coordinate; α, heat-transfer coefficient; α_*, dispersion parameter; β, volume gas content; Δi, enthalpy difference; T, $\bar{T}$, temperature, relative temperature; t, temperature pulsation; T_W, wall temperature; $\sqrt{t'^2}$, mean-square temperature pulsation; ΔT, temperature difference; δ_T^{**}, relative energy-loss thickness; η_*, effective dissipation coefficient; $\varkappa$, $\varkappa_1$, coefficients in linear displacement path-length formulas; λ', liquid-phase thermal conductivity; μ', liquid-phase dynamic viscosity; ν', liquid-phase kinematic viscosity; ξ, $\xi^{**} = y/\delta^{**}$, relative transverse coordinate; ρ, density at a given point; ρ_0, density of unperturbed flow; $\tilde{\rho} = \rho/\rho_0$, relative density $\tilde{\rho}'' = \rho''/\rho'$, relative phase density; $\psi = \rho_0/\rho_W$; σ, surface tension; τ, tangential stress; τ_W, tangential stress at wall; $\tilde{\tau} = \tau/\tau_W$, relative tangential stress; $\tilde{\tau}$, characteristic time; $\varphi = \delta_1/\delta_0$, relative surface perturbation; Ψ, relative friction coefficient; ω, relative flow velocity at given point of boundary layer; $b = \rho_W u_W 2/\rho_0 U_0 c_{f_0}$; $b_T = \rho_W u_W/\rho_0 u_0 St_0$, permeability parameter; b_{cr}, critical injection parameter; ε, ε_0, turbulence intensity in boundary layer and external flow N, number of events K, wave number $\lambda_0 = -(\delta/u_0)(du_0/dx)2/c_{f_0}$, parameter characterizing longitudinal pressure.

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